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Формула для прогиба решетчатой фермы, имеющей случаи кинематической изменяемости

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Аннотация
Плоская ферма с двумя неподвижными шарнирными опорами внешне статически неопределима. Рассмотрено действие вертикальной нагрузки равномерно распределенной по узлам верхнего пояса. Выводится формула для прогиба фермы. Замечено, что при нечетном числе панелей определитель системы уравнений равновесия обращается в ноль, что соответствует мгновенной изменяемости конструкции. Методом индукции формула для прогиба обобщается на произвольное число панелей. Использована система символьной математики Maple.
Ключевые слова: ферма, прогиб, метод индукции, Maple

The formula for deflection of truss with cases of kinematic variability

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Abstract
A flat truss with two fixed hinged supports is externally statically indeterminate. The effect of a vertical load uniformly distributed over the nodes of the upper belt is considered. A formula is derived for the deflection of the truss. It is noted that for an odd number of panels, the determinant of the system of equations of equilibrium turns to zero, which corresponds to the instantaneous variability of the structure. By induction, the deflection formula is generalized to an arbitrary number of panels. The system of symbolic mathematics Maple was used.
Keywords: truss, deflection, induction method, Maple.

The lattice truss with two supports (Figure 1) is externally statically indeterminated. To determine the forces in the rods and the reactions of support, it is necessary to compile a system of equations of equilibrium for all nodes at once. In a farm with *n* panels in, counting on the lower belt, the number of rods m=8(n+1) including four support rods (two bars per support).

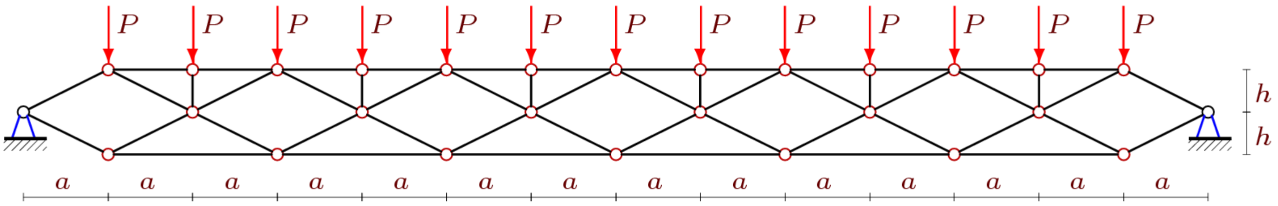

Figure 1 — Truss at n=3k=6

To determine the deflection by the Maxwell-Mora formula, it is necessary to know the forces in the rods. The solution in symbolic form will be obtained according to the program [1]. To do this, it is needed to number the rods and knots (Figure 2). The origin is located in the left fixed support.

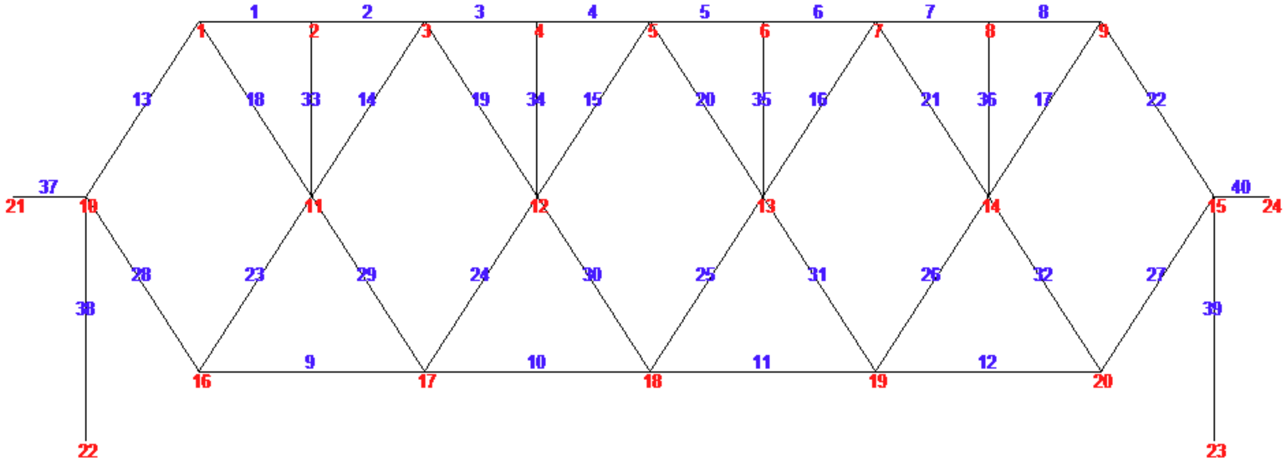

Figure 2 – Numbering of nodes and rods, n = 2k = 4

Here is the fragment of input of the coordinate in the program:

```
>for i to 2*n+1 do x[i]:=(i-1)*a; y[i]:=2*h; od:
>for i to n+2 do x[i+2*n+1]:=(i-1)*2*a-a; y[i+2*n+1]:=h; od:
>for i to n+1 do x[i+3*n+3]:=(i-1)*2*a; y[i+3*n+3]:=0; od:
```

The coordinates of the nodes determine the direction cosines of the system of equilibrium equations written in the matrix form. The solution is obtained using the inverse matrix method, which is easily implemented in the Maple system and significantly faster than the specialized operators of the Linear Algebra package of the same system. To calculate the deflection of the truss, we use the Maxwell-Mora formula in the form:

$$\Delta = \sum_{i=1}^{m-4} S_i s_i l_i / (EF) \qquad (1)$$

where E — the modulus of elasticity of the rods, F — the cross-sectional area of the rods (the same for the entire structure), l_j and S_j — the length of the j-th rod and the force in it from the action of a given load, s_j — the force from a single vertical force applied to the middle of the span in the upper belt. The summation is carried out over all the rods of the farm, except for the support ones, which are assumed to be rigid.

Analysis of a series of solutions for trusses with a number of panels from 1 to 20 showed that the deflection formula has the same form

$$EF\Delta = P(A_k a^3 + C_k c^3)/h^2. \qquad (2)$$

Using the **rgf_findrecur** operator of the Maple system, it turns out that the sequence of coefficients a^3 satisfies a homogeneous linear recurrence equation of the fifth order

$$A_k = 5A_{k-1} - 10A_{k-2} + 10A_{k-3} - 5A_{k-4} + A_{k-5}.$$

The solution of this equation with the initial conditions

$$A_1 = 8, A_2 = 64, A_3 = 248, A_4 = 680, A_5 = 1520$$

is a function $A_k = k(k^3 + 10k^2 + 7k + 2)/3$. Similarly, from another equation $C_k = 3C_{k-1} - 3C_{k-2} + C_{k-3}$ we have the solution $C_k = k(1+k)$. Thus, the solution of the problem of the deflection of a truss is obtained for any number of panels.

In Fig. 3 shows the solution curves (2) for the dimensionless deflection $\Delta' = EF\Delta/(P_{sum}L)$ calculated at a constant span length of the truss L = 100 m, $a = L/(n+1)$ and the total load $P_{sum} = (2n+1)P$.

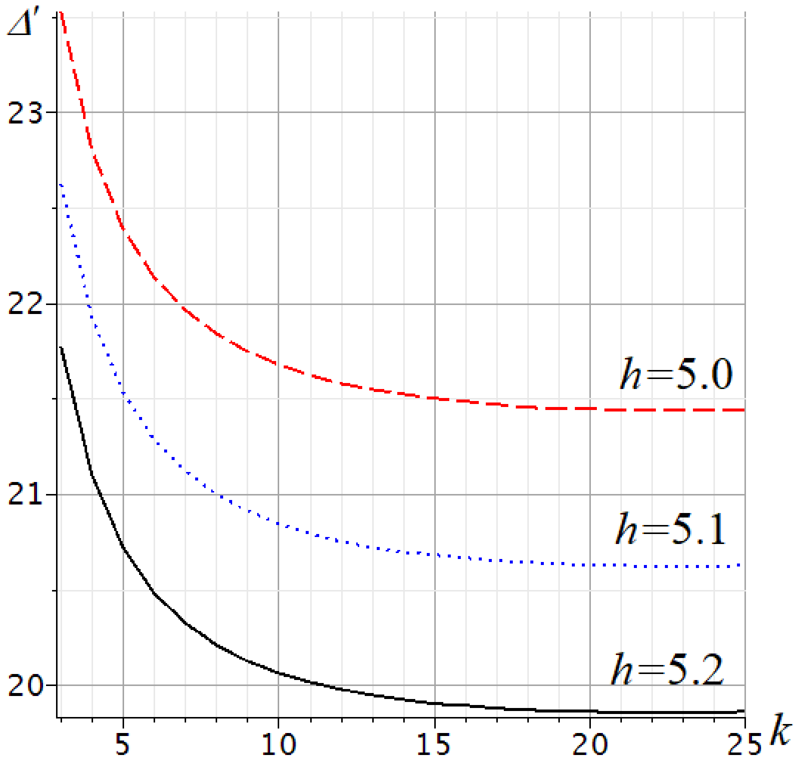

Figure 3 – Dependence of the deflection on the number of panels, height *h* in meters

The truss is solved for an even number of panels, but the attempt to obtain a solution with an odd number was unsuccessful. The determinant of the system of equations turned to zero. This indicates a kinematic changeability of the system.

A truss with a similar lattice, but with one fixed support and one movable is analytically calculated in [2]. The method of induction, which makes it possible to obtain exact solutions for an arbitrary number of panels, was previously used in solving problems of plane [3-8] and spatial trusses [9, 10]. A brief review of the work on this topic is contained in [11]. The problems of statically determinate regular trusses are discussed in [12].

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