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Условие разрушения сжатых стержней из реологических материалов при выпучивании

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Аннотация
Материал стержня подчиняется нелинейным соотношениям ползучести с учетом структурного параметра, отвечающего за степень растрескивания при разрушении. За критерий выпучивания принято условие бифуркации прогиба при нулевой скорости его возмущения. Рассмотрен шарнирно опертый стержень, сжатый постоянной во времени продольной силой.
Ключевые слова: ползучесть, разрушение, выпучивание, стержень, нелинейность

The condition for the destruction of compressed rods from rheological materials during buckling

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Abstract
The material of the rod obeys the nonlinear creep relations and takes into account the structural parameter responsible for the degree of cracking during fracture. For the buckling criterion, the bifurcation condition of the deflection at the zero velocity of its perturbation is assumed. A pivotally supported rod, compressed by a constant longitudinal force, is considered.
Keywords: creep, failure, buckling, rod, nonlinearity.

One of the important practical problems, the solution of which is available in the framework of the mechanics of a deformable solid, is the identification of the conditions for the destruction of structural elements. In [1-3], various approaches to this problem are described: the modeling of deformation with the introduction of an additional parameter in the determining ratio, monotonously increasing with the loss of strength of the material, analysis of buckling and fiber stratification, etc. In [4,5], the destruction and buckling is associated with the instability of the material.

Let the material of the rod obey the nonlinear kinetic relationship of creep with hardening [1]

$$h(p)\dot{p} = f(\sigma)/(1-\omega). \quad (1)$$

Here $p = \varepsilon - \sigma / E$ is the creep strain, $h(p)$ is the hardening function, ω is the structural parameter, $0 < \omega < 1$. At the beginning of deformation $\omega = 0$ (undamaged material). When material is destroyed $\omega = 1$. Suppose that for the structural parameter (the damaging parameter) the following kinetic determining relation is valid [1]

$$\dot{\omega} = g(\sigma) / (1 - \omega). \tag{2}$$

The function $g(\sigma)$ is determined from the experiment. For a rod compressed by an axial force (Fig. 1), we have relations

$$\int \sigma dF = -T, \int \Delta\sigma z dF = -T\Delta v, \int \Delta\varepsilon z dF = J\Delta v_{,yy}, \tag{3}$$

where $J = \int z^2 dF$ is the moment of inertia of the section, $v_{,yy} = d^2 v / dy^2$. The symbol Δ indicates a small increment of the corresponding quantity.

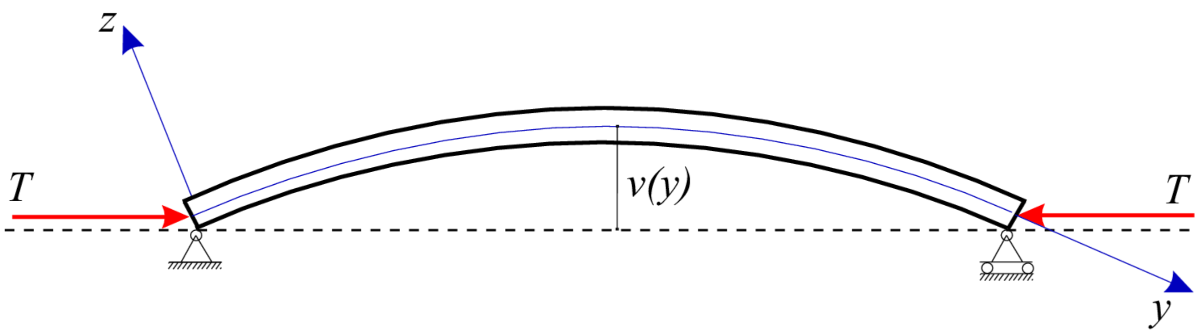

Figure 1

The variation of the defining relation of the medium (1) gives the following equation for transformations

$$\Delta\dot{p}h + \Delta p h' = f'\Delta\sigma + f\Delta\omega / (1 - \omega)^2. \tag{4}$$

We multiply (4) by z and integrate over the area F

$$h\int_F \Delta\dot{p}\, z\, dF + \dot{p}h' \int_F \Delta p z\, dF = f' \int_F \Delta\sigma z\, dF + f\int_F \Delta\omega z\, dF / (1 - \omega)^2. \tag{5}$$

From the variation (2) it follows that

$$\Delta\omega = (1/\dot{\omega})(\Delta\dot{\omega}(1 - \omega) - g'\Delta\sigma). \tag{6}$$

We divide both sides of equation (5) by h and taking into account (6), and also that the values without the signs Δ do not depend on z, with the help of equations (2) we obtain

$$J\Delta\dot{v}_{,yy} + T\Delta\dot{v} / E + \dot{p}(T\Delta v / E + J\Delta v_{,yy})h' / h = -T(\Delta v / h)(f' - \varphi) + (\dot{p} / \dot{\omega})\int \Delta\dot{\omega} z dF.$$

where

$$\varphi = f(\sigma)g'(\sigma)/(\dot{\omega}(1-\omega)^2) = fg'/(g(1-\omega)).$$

If we choose the shape of the deflection in the form of a sinusoid $\Delta v = U_0 \sin \mu y,$ then the deflection velocity should have the same form $\Delta \dot{v} = U_1 \sin \mu y,\ \Delta \dot{v}_{,yy} = -\mu^2 U_1 \sin \mu y.$ Let the increment of the parameter ω have a distribution along the thickness of the section of the form $\Delta\omega = \Omega \sin \mu y,\ \Delta\dot{\omega} = \dot{\Omega} \sin \mu y.$ We have from the last equation:

$$(-\mu^2 J + T/E)U_1 + \dot{p}(T/E - \mu^2 J)U_0 h'/h = -T(U_0/h)(f' - \varphi) + (\dot{p}/\dot{\omega})\int \dot{\Omega} z dF.$$

Obviously, the disturbances received by the system before the moment of destruction (buckling) immediately after the perturbation, decrease at least for some time. If the perturbations are attached to the system after a critical time, then they immediately grow. Assuming that at the time of destruction, the increment of the velocities is zero $U_1 = 0,\ \dot{\Omega} = 0$, we obtain

$$\dot{p}(T/E - \mu^2 J)U_0 h' = -TU_0(f' - \varphi).$$

Critical load of the elastic rod $T^* = EJ\mu^2$ [4]. We divide the last equality by $-\mu^2 J + T/E$ and obtain

$$U_0(\dot{p}h' - Ek(f' - \varphi)/(1-k)) = 0,$$

where $k = T/T^*$. From the condition $U_0 \neq 0$ we obtain the desired relation for the critical values corresponding to the destruction of the rod at the moment of buckling

$$\dot{p}h' - Ek(f' - \varphi)/(1-k) = 0. \qquad (7)$$

If $\varphi = 0$ the equation obtained has the form $h'/h - Ek(f'/f)/(1-k) = 0$ and coincides with the condition of pseudo-furcation [6] and a singular point of the first order [4, 5, 7]. By dividing (7) into $h = f/(\dot{p}(1-\omega))$, from the critical relation (7), we can take the creep rate and rewrite it in the form

$$\frac{h'}{h} = \frac{Ek}{1-k}(f'/f)(1-\omega) - g'/g).$$

By artificially raising the order of the determining relationship and adding equilibrium equations differentiated by time [4] to the system of equations, it is possible to analyze other critical situations in the material when the bifurcation occurs when higher-order derivatives of perturbed functions.

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